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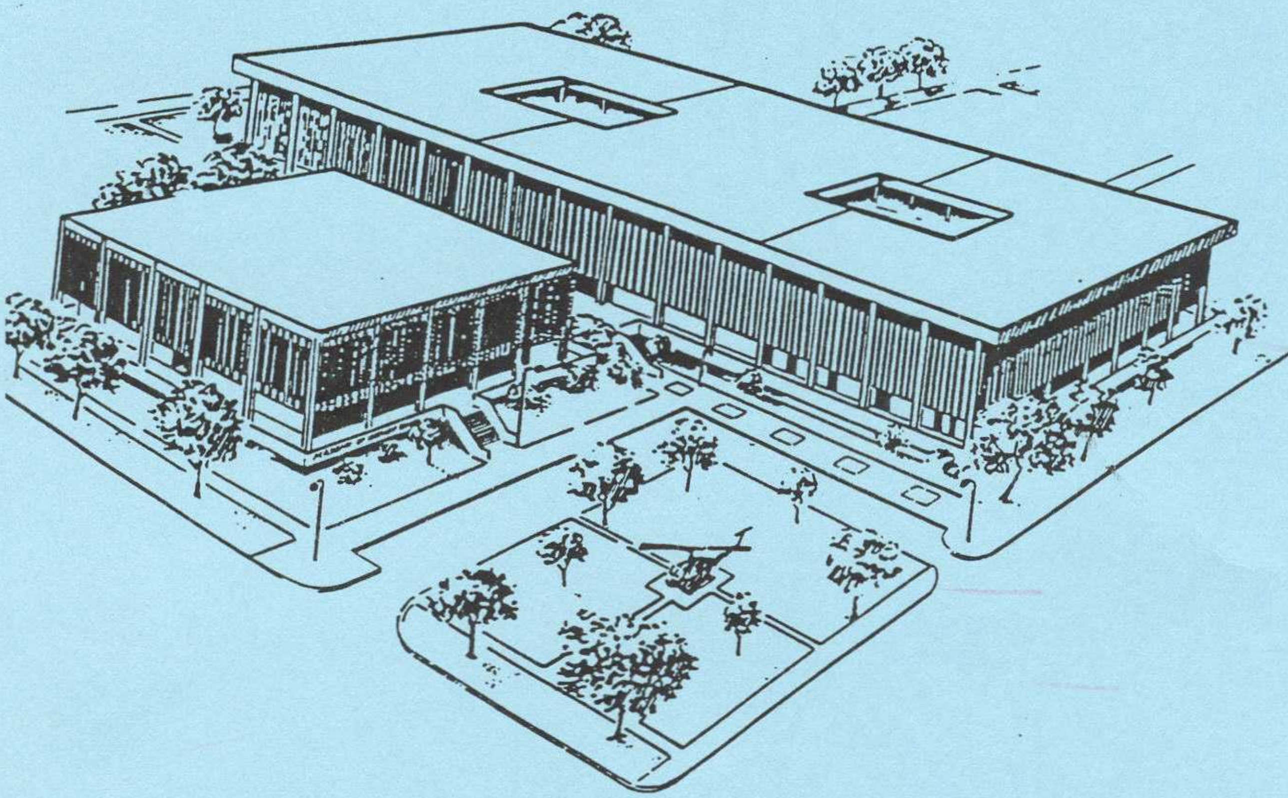

MATHEMATICS REVIEW

6F-66F

ANESTHESIOLOGY FOR ARMY NURSE CORPS OFFICER

MARCH 1994

TABLE OF CONTENTS

ACADEMY OF HEALTH SCIENCES, U.S. ARMY
DEPARTMENT OF CLINICAL SUPPORT SERVICES
CHEMISTRY BRANCH

LABORATORY MATHEMATICS REVIEW - 6F-66F ANESTHESIOLOGY FOR ARMY NURSE CORPS OFFICERS

INTRODUCTION

This mathematics review was developed to prepare you in mathematical skills for the 6F-66F Anesthesiology For Army Nurse Corps Officer Course. The emphasis is upon computations related to solutions and their concentrations. If you feel that you need a more basic review of mathematics before completing this review, you should refer to APPENDIX H on basic mathematics, which covers addition, subtraction, multiplication, and division of whole numbers; decimals and fractions; and conversions to and from the metric system. If you need a basic review of chemistry, you should refer to General Chemistry review handout.

You must be able to solve the types of problems included in the lesson exercises if you are to perform well in the 6F-66F course.

Mathematics Review Components:

This review consists of six lessons, and Appendices A through H. The lessons are:

Lesson 1, General Mathematics Review.

Lesson 2, Introduction to Solution Mathematics.

Lesson 3, Molar Solutions.

Lesson 4, Equivalent Solutions.

Lesson 5, Conversion of Concentration Units.

Lesson 6, pH and Buffers.

Lesson Materials Furnished:

Lesson materials provided in this booklet include the 6 lessons, and exercises and solutions for all lessons in this booklet.

Procedures for Math Review Completion:

You are encouraged to complete the math review lesson by lesson. When you have completed all of the lessons to your satisfaction, answer the questions to all the exercises. Be prepared to take an examination on this material soon after you arrive for the 6F-66F course.

Study Suggestions:

Here are some suggestions that may be helpful to you in completing this mathematics review:

--Read and study each lesson carefully.

--Complete the review lesson by lesson. After completing each lesson, work the exercises at the end of the lesson, marking your answers in this booklet. (NOTE: In lesson 1 of this review, the exercises are at the end of each section rather that at the end of the lesson.)

--After completing each set of lesson exercises, compare your answers with those on the solution sheet which follows the exercises. If you have answered an exercise incorrectly, check the reference cited after the answer on the solution sheet to determine why your response was not the correct one.

--As you successfully complete each lesson, go on to the next.

--Since one topic in mathematics often builds upon previous topics or lessons, you must often apply in later lessons what you have already learned in earlier lessons.

LESSON ASSIGNMENT SHEET

LESSON 1	--General Mathematics Review.
LESSON ASSIGNMENT	--Paragraphs 1-1 through 1-39.
LESSON OBJECTIVES	--After completing this lesson, you should be able to:

1-1. Identify applications of the properties of real numbers and perform related computations.

1-2. Use properties of equality to solve equations for unknowns.

1-3. Express numbers in standard scientific notation and perform arithmetic calculations using standard scientific notation.

1-4. Find the logarithm for a given number, find the antilogarithm for a given logarithm, and perform arithmetic calculations using logarithms.

SUGGESTION --After reading each section, complete the exercises at the end of each section. These exercises will help you to achieve the lesson objectives.

LESSON 1

GENERAL MATHEMATICS REVIEW

Section I. PROPERTIES OF REAL NUMBERS

1-1. DISCUSSION

Real numbers can be described as being the set of all x, such that x is the coordinate of a point on a number line. The real numbers include the rational and irrational numbers. Rational numbers include integers and quotients of integers. Integers are whole numbers. Positive integers, such as 1, 2, 3, and so forth are also called natural numbers. An operation is a method of combining two elements of the set of real numbers to form a third element (not exclusive). The properties of real numbers depend upon two basic operations, addition and multiplication. This section is designed to give you the concepts and skills necessary for the content and course work to follow.

1-2. SOME FUNDAMENTAL PROPERTIES OF REAL NUMBERS

For any real numbers a, b, c, and d, the additive and multiplicative properties are as follows:

a. **Commutative Properties.**

$a + b = b + a$ $\qquad$ $ab = ba$

Examples:

$3 + 5 = 5 + 3$ $\qquad$ $2 \times 4 = 4 \times 2$

b. **Associative Properties.**

$(a + b) + c = a + (b + c)$ $\qquad$ $(ab)c = a(bc)$

Examples:

$(2 + 1) + 4 = 2 + (1 + 4)$ $\qquad$ $(4 \times 3) \times 2 = 4 \times (3 \times 2)$

c. **Identity Properties.**

$a + 0 = a$ $\qquad$ $a \times 1 = a$

Examples:

$3 + 0 = 3$ $\qquad$ $7 \times 1 = 7$

d. Inverse Properties.

$a + (-a) = 0$

For each nonzero real number a, there is a unique real number 1/a such that:

$\frac{1}{a} \times a = 1$

Examples:

$3 + (-3) = 0$

$\frac{1}{12} \times 12 = 1$

e. Distributive Property.

$a\,(b + c) = ab + ac$

Example: $2(3 + 2) = (2 \times 3) + (2 \times 2)$

1-3. DEFINITIONS OF SUBTRACTION AND DIVISION

a. Subtraction is a Special Case of Addition.

$a - b = a + (-b)$

Example: $5 - 2 = 5 + (-2)$

b. Division is a Special Case of Multiplication.

$\frac{a}{b} = a \times \frac{1}{b}$ provided that b is not equal to 0

Example: $\frac{6}{3} = 6 \times \frac{1}{3}$

NOTE: Division by 0 is undefined, because any real number divided by 0 has no solution.

1-4. PROPERTIES OF ZERO (0)

a. $a \times 0 = 0$

Example: $5 \times 0 = 0$

b. If $\frac{a}{b} = 0$, then $a = 0$

Example: $\frac{0}{3} = 0$

1-5. PROPERTIES OF NEGATIVE NUMBERS

a. $(-a)\ (-b) = ab$

Example: $(-3)\ (-4) = 12$

<u>NOTE</u>: When no arithmetic operator (+, -, ÷) is present between two real numbers, one or both being placed in parentheses, multiplication is implied.

b. $(-a)\ b = -\ (ab)$

Example: $(-2)3 = -6$

c. $-a = (-1)a$

Example: $-13 = (-1)13$

1-6. PROPERTIES OF FRACTIONS

a. $\frac{a}{b} = \frac{c}{d}$ if and only if $ad = bc$

Example: $\frac{4}{2} = \frac{8}{4}$ $\quad (4)\ (4) = (2)\ (8)$

b. $\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}$

Example: $\frac{6}{2} + \frac{4}{2} = \frac{6 + 4}{2}$

c. $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$

Example: $\frac{4}{3} + \frac{1}{2} = \frac{(4)(2) + (3)(1)}{(3)(2)}$

d. $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

Example: $\frac{2}{3} \times \frac{6}{5} = \frac{12}{15}$

e. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$

Example: $\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4}$

f. $-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$

Example: $-\frac{6}{3} = \frac{-6}{3} = \frac{6}{-3} = -2$

1-7. ORDER OF OPERATIONS

a. If parentheses or brackets are present, evaluate the expressions within first, starting with the innermost set of parentheses using the rules that follow.

b. Evaluate any exponential expressions.

c. Evaluate any multiplications or divisions from left to right.

d. Evaluate any additions or subtractions from left to right.

e. If an expression is in fractional form, perform stated operations in the numerator and denominator, then simplify if possible.

Examples:

$\frac{3 + 10}{2 - 4} = -\frac{13}{2}$ $\qquad \frac{15}{9 - 6} = 5$

<u>NOTE</u>: A good way to remember the order of operations is to use the memory aid, "Please excuse my dear Aunt Sally." The order for solving algebra equations is: parenthesis, exponents, multiplication and division, addition and subtraction.

1-8. EXERCISES, SECTION I

After you have completed these exercises, turn to the end of the lesson, and check your answers with the review solutions.

FIRST REQUIREMENT: Identify the property of real numbers that justifies each of the following mathematical statements:

a. $2 + (p - 1) = (2 + p) - 1$

____

b. $4 + a = a + 4$

____

c. $\frac{1}{7 + e}(7 + e) = 1$

____

d. $3 \times 1 = 3$

____

e. $r(1 + 8) = r + 8r$

____

f. $-11 + 11 = 0$

____

g. $\tilde{5} + 0 = 5$

____

h. $12 - 6 = (-6) + 12$

____

i. $3(2 - 5) = 6 - 15$

____

j. $v - v = 0$

____

SECOND REQUIREMENT: Evaluate the following:

k. $-6(-3)$

____

l. $-12 - 3$

____

m. $\frac{15}{3} + 4 + 7$

____

n. $\frac{24}{8 - 6}$

____

o. $\frac{2}{5} - \frac{5}{3}$

____

p. $6(30Y + 4)$

____

q. $4 + 2 \times 4 - \frac{9}{3}$

____

r. 150×0

____

s. $\frac{2}{9} + \frac{5}{9}$

____

t. $\frac{2}{8} - \frac{4}{8}$

____

Section II. PROPERTIES OF EQUALITY

1-9. DISCUSSION

An equation is a statement that two expressions or quantities are equal in value. The statement of equality $2x + 3 = 11$ is an equation. It is algebraic shorthand for "the sum of two times some number plus three is equal to eleven." Of course this algebraic shorthand makes problem solving much easier. Solving an equation is determining the values of the unknown numbers in that equation that makes the equation true. Various techniques may be employed in the solving of equations. The form of the equation, and the degree (highest power a variable is raised in the equation) will determine the method utilized in problem solving. Most of the problem solving in the following course work will involve first-degree equations; that is equations whose variables are raised to a power of 1 (one), so the following methods will develop techniques to solve equations of this type.

NOTE: Some second-degree equations will be encountered in the pH and buffers problems. However, methods taught here will be sufficient for any problem solving.

1-10. PROPERTIES OF EQUALITY

For any real number a, b, or c, the properties are as follows.

a. **Reflexive Property.**

$a = a$

Example: $8 = 8$

b. **Symmetric Property.**

If $a = b$, then $b = a$

Example: If $x + 5 = y$, then $y = x + 5$

c. **Transitive Property.**

If $a = b$ and $b = c$, then $a = c$

Example: If $a + b = c$, and $c = 3a$, then $a + b = 3a$

d. **Substitution Property.**

If $k = x$, then k may replace x in any equation without changing the truth of the equation.

Example: If $2k - 4 = x$, and $x + k = 9$, then $2k - 4 + k = 9$

1-11. RULES OF EQUALITY

a. Adding or subtracting the same quantity on both sides of an equation produces an equivalent equation.

b. Multiplying or dividing both sides of an equation by the same nonzero quantity produces an equivalent equation.

c. Simplifying an expression on either side of an equation produces an equivalent equation.

1-12. SOLVING FIRST DEGREE EQUATIONS

a. Eliminate any fractions by multiplication.

b. Simplify each side of the equation as much as possible by combining like terms.

c. Use the addition property to simplify the equation so that all terms with the desired variable are on one side of the equation and all numbers and variables other than the one being solved for are on the other side.

d. Use multiplication or division to get an equation with just the desired variable on one side.

e. Check your answer by substituting the solution back into the original equation and evaluating it for truth when practical.

1-13. PROBLEM-SOLVING TECHNIQUES

a. **Solve for x:**

$3[1 - 2(x + 1)] = 2 - x$

$3[1 - 2x - 2] = 2 - x$ simplify

$3(-1 - 2x) = 2 - x$

$-3 - 6x = 2 - x$

$-3 - 5x = 2$	add x to both sides
$-5x = 5$	add 3 to both sides
$x = -1$	divide both sides by -5

Check the Solution.

$3[1 - 2(-1 + 1)] = 2 - (-1)$	substitute -1 for all x
$3[1 - 2(0)] = 2 - (-1)$	evaluate
$3(1) = 2 + 1$	true
$3 = 3$	

b. **Solve for x:**

$$\frac{x}{2} - \frac{3x}{4} = 1$$

$4\boxed{\frac{x}{2} - \frac{3x}{4}} = (4)1$	multiply both sides by the least common denominator
$2x - 3x = 4$	simplify
$-x = 4$	
$x = -4$	multiply both sides by -1

Check the Solution.

$\frac{(-4)}{2} - \frac{3(-4)}{4} = 1$	substitute -4 for all x
$-2 + 3 = 1$	true
$1 = 1$	

c. **Solve for y:**

$9x + abc = yz - mno$	
$9x + abc + mno = yz$	adding mno to both sides
$\frac{9x + abc + mno}{z} = y$	dividing both sides by z

NOTE: With problems that contain more than one variable, it is not practical to check for truth. However, double check your work for accuracy.

1-14. EXERCISES, SECTION II

After you have completed these exercises, turn to the end of the lesson, and check your answers with the review solutions.

a. $1 - (4c + 7) = -5c$

____

b. $8 + 6b = 3b - 4$

____

c. $r - (5 - [r - (4 + r)]) = 8$

____

d. $8z - 2z + 6 = z - 6$

____

e. $-\frac{5k}{9} = 2$

____

f. $4(v - 1) - 4(2v + 2) = 8$

____

g. $6r - \frac{9r}{2} = 6$

____

h. $\frac{7r}{2} - \frac{9r}{2} = 16$

____

i. $\frac{y - 8}{5} + \frac{y}{3} = \frac{-8}{5}$

____

j. Solve for y:

$4x + 6y = 12$

k. $-7 - 5k = 10k - 7$

l. $P + 9 = 0$

m. $-7X + 1 = 50$

n. $3(x + 5) = 2x - 1$

o. $2 + \frac{m}{3} = \frac{m}{5}$

p. $5 - 8k = -19$

q. Solve for z:

$abz = pqr$

r. Solve for c:

$rst = abc - ghi$

s. Solve for x:

$\frac{abc}{def} = \frac{rst}{xyz}$

t. Solve for b:

$b + c = 0$

Section III. SCIENTIFIC NOTATION

1-15. DISCUSSION

In the clinical laboratory, elementary mathematics are frequently used by a laboratory specialist to calculate the amount of a specific chemical needed to prepare a reagent or to determine the concentration of a chemical constituent in a clinical sample. To enable a laboratory specialist to perform calculations with greater efficiency, the use of scientific notation must be mastered.

1-16. EXPONENTS

An exponent is a superscript number written to the right of a base number. An exponent indicates the number of times the base number is to be used as a multiplicative factor to produce a product equal to the exponential expression.

a. **General Examples.**

(1) $3^2 = 3 \text{ X } 3 = 9$

In the above example, the exponent is the number two (2) and indicates that the number three (3), referred to as the base number, is to be used as a multiplicative factor twice.

(2) $2^3 = 2 \text{ X } 2 \text{ X } 2 = 8$

In the above example, the exponent is the number three (3) and indicates that the base number two (2) is to be used as a multiplicative factor three times.

b. **Examples Using Powers of Ten (10).**

(1) $10^2 = 10 \text{ X } 10 = 100$

(2) $10^3 = 10 \text{ X } 10 \text{ X } 10 = 1000$

1-17. RULES OF EXPONENTIATION

The following rules apply to exponential expressions that have the same base with the exception of addition and subtraction.

a. **Addition and Subtraction.** The expressions must first be evaluated. Then, addition or subtraction is performed in the usual manner.

Example.

$10^2 + 10^1 = 100 + 10 = 110$

b. **Multiplication.**

$x^a x^b = x^{a+b}$

Example.

$2^3 2^2 = 2^{3+2} = 2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

c. **Division.**

$\frac{x^a}{x^b} = x^{a-b}$

Example.

$\frac{2^4}{2^2} = 2^{4-2} = 2^2 = 2 \times 2 = 4$

d. **Exponential Expressions Raised to a Power.**

$(x^a)^b = x^{ab}$

Example.

$(2^2)^3 = 2^{(2)(3)} = 2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

1-18. EXPRESSING NUMBERS IN TERMS OF SCIENTIFIC NOTATION

a. **Division.** Scientific notation uses the exponential method of expressing very large and very small numbers in terms of an exponential expression. Utilizing this method, numbers are expressed as a product of two numbers. The first number is called the digit term and is a number greater than or equal to one (1) but less than ten (10). The second number of the product is called the exponential term and is written as a power of ten (10).

b. **Examples.**

(1) $100 = 1 \times 10 \times 10 = 1 \times 10^2$

In the above example, the digit term is the number one (1) and the exponential term is ten squared (10^2).

(2) $200 = 2 \times 10 \times 10 = 2 \times 10^2$

In this example, the digit term is the number two (2) and the exponential term is ten squared (10^2).

(3) $745 = 7.45 \times 10 \times 10 = 7.45 \times 10^2$

(4) $0.00486 = 4.86 \times 10^{-1} \times 10^{-1} \times 10^{-1} = 4.86 \times 10^{-3}$

c. **Decimal Movement.** In examining the above examples, it is easily seen that the exponent of the number ten (10) is determined by movement of the decimal point. The exponent of the number ten (10) is positive when the decimal is moved to the left and negative when moved to the right. Standard scientific notation refers to the placement of the decimal to the right of the first non-zero integer.

(1) Positive exponents. When the decimal point is moved to the left, the exponent of the number ten (10) is always positive.

(2) Negative exponents. When the decimal point is moved to the right, the exponent of the number ten (10) is always negative.

(3) Examples.

$214 = 2.14 \times 10^2$

$0.102 = 1.02 \times 10^{-1}$

In the first example using the number 214, the decimal was moved two places to the left and thus the exponent of the number ten (10) is two (2). In the second example using the number 0.102, the decimal was moved one place to the right and thus the exponent of the number ten (10) is minus one (-1).

d. **Powers of Ten (10) Expressed in Scientific Notation.**

$1{,}000{,}000 = 1 \times 10^6$	
$100{,}000 = 1 \times 10^5$	
$10{,}000 = 1 \times 10^4$	
$1{,}000 = 1 \times 10^3$	decimal moved to the left
$100 = 1 \times 10^2$	
$10 = 1 \times 10^1$	
$1 = 1 \times 10^0$	------------------------
$0.1 = 1 \times 10^{-1}$	
$0.01 = 1 \times 10^{-2}$	
$0.001 = 1 \times 10^{-3}$	
$0.0001 = 1 \times 10^{-4}$	decimal moved to the right
$0.00001 = 1 \times 10^{-5}$	
$0.000001 = 1 \times 10^{-6}$	

1-19. CHANGING THE SIGN OF THE EXPONENT

To change the sign of the exponent, move the exponential term to the denominator or numerator as appropriate with a concurrent change in sign. *This procedure yields* an equivalent expression.

a. **Example.**

$$\begin{array}{ll}\text{Numerator} & \dfrac{10^{-6}}{10^{4}} = \dfrac{10^{-4}}{10^{6}} = 10^{-10} \\ \text{Denominator} & \end{array}$$

When an exponential term is moved from one side of the division line to the other, the exponent changes signs.

b. **Example.** Write and equivalent expression of 10^2, changing the sign of the exponent.

Solution.

$$\begin{array}{ll}\text{Numerator} & \dfrac{10^{2}}{1} = \dfrac{1}{10^{-2}} \\ \text{Denominator} & \end{array}$$

c. **Example.** Write an expression that changes the sign of the exponential term in $1/10^6$ without changing the value.

Solution.

$$\frac{1}{10^6} = \frac{10^{-6}}{1} = 10^{-6}$$

d. **Example.** Write an expression that changes the signs of the exponential without changing the value.

$$\frac{4.8 \text{ X } 10^{-7}}{1.3 \text{ X } 10^{4}}$$

Solution.

$$\frac{4.8 \text{ X } 10^{-7}}{1.3 \text{ X } 10^{4}} = \frac{4.8 \text{ X } 10^{-4}}{1.3 \text{ X } 10^{7}} = 3.7 \text{ X } 10^{-11}$$

1-20. MULTIPLICATION OF NUMBERS EXPRESSED IN SCIENTIFIC NOTATION

To multiply numbers expressed in scientific notation, multiply the digit terms in the usual way and add the exponents of the exponential terms.

a. **Example.** Multiply 10^2 by 10^3

Solution. In multiplying exponential terms, the exponents of the base ten (10) are added (the base numbers must be the same), therefore: